MADE EASY

Computer Science Engineering / IT

Toppers Handwritten Notes

Discrete Mathematics

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- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

15-Sept

DISCRETE MATHEMATICS

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9 Marks

Chapters:

1. Logic
2. Combinatorics
3. Set Theory [KOLMAN, BUSAN $ ROSS]
4. Graph Theory [NARSINGH DEO] [Theory]

LOGIC

1. Logical Statement ? [Proposition]
2. Logical Operators & their properties ($\vee$, $\wedge$, $\neg$, $\Rightarrow$, $\Leftrightarrow$, $\oplus$ $\uparrow$ $\downarrow$) — Disjunction, Conj., Negation
3. Tautology (T), Contradiction (C) & Contingency (CT) [Satisfiable (T or CT) / Unsatisfiable (C)]
4. Normal Forms & properties: PDNF (Principle Disjunctive Normal Form), PCNF (Principle Conjunction Normal Form)
5. Implications $ Biconditional ($\Rightarrow$, $\Leftrightarrow$)
6. Arguments & Fallacy [Invalid Argument]
7. Rules of Inference
8. Predicate Logic - Quantifier ($\forall$, $\exists$)
   - Validity of a predicate
   - Properties
   - Translation

LOGIC

(S, o)
↓ ↳ operators
Set of all Logical Stmt

ogical StateMent- (Preposition)

- Declarative sentence which can be either true or false but not both.

Ex - This board is white.
This Fan is Rotating.
- This sentence is true.

[is/will tends to declaration]

ta logical StateMent

1. Questions - What is Your Name?
2. Command - Stand up.
3. Exclamation - Oh! That's great.
4. $x+2=4$
   (it is not preposition bcoz for some x value it is true
   it is false)
5. He is tall. (unless he is specified)
6. Today is Wednesday.
   [Not a preposition bcoz today may be true, but tommorow it will become false]
7. Tommorow it will rain.
   [Not a preposition.]
8. This sentence is false.
   [Negative Self Referential Sentence]

ogical operators:

A preposition is written in the following way:

$p: 2+2=4$

$q(x): x+2=4$ (Predicate) but not a preposition

~~True~~ False — $\forall x\, P(x)$
True — $\exists x\, P(x)$
] prepositions

gation- ($\sim$, $\bar{p}$, $\neg$, p') · Unary operators

P	$\bar{P}$
0	1
1	0

P	Negation
is	is not
is not	is
=	≠
<	≥
>	≤
$p \vee q$	$p' \wedge q' \Rightarrow p \downarrow q$
$p \wedge q$	$p' \vee q' \Rightarrow p \uparrow q$

P	Negation
$p \Rightarrow q$	pq'
$p \Leftrightarrow q$	$p \oplus q$
$p \oplus q$	$p \Leftrightarrow q$
$p \uparrow q$	$p \wedge q$
$p \downarrow q$	$p \vee q$

- if $p \vee q = 1$
  than $p = \sim q$ is one possibility but not the sure thing. it also allow some other thing
- if $p \wedge q = 0$
  $\Rightarrow [p = \sim q]$ not allways.
- If $p \vee q = 1$ & $p \wedge q = 0 \Rightarrow \boxed{p = \sim q}$
- If $p \Rightarrow 2+2=4$ or $3+7=10$
  $\bar{p} \Rightarrow$ ~~2~~ $2+2 \neq 4$ and $3+7 \neq 10$
- If $p \Rightarrow 2+2=4$ and $3+7=10$
  $\bar{p} \Rightarrow 2+2 \neq 4$ OR $3+7 \neq 10$
- p: 2 is even & divisible by 4.
  p': 2 is odd or not divisible by 4.
- p: if it rains, i will carry umbrella. [Either it does not rain OR I will carry umbrella]
  p': It rains and I will not carry umbrella

$\wedge$ - all must be true
$\vee$ - atleast one is true
$\oplus$ - Exactly one is true
$\uparrow$ - Atleast one is false

Conversion of Secondary operators into Basic operators:

- $p \rightarrow q = p' + q$
- $p \Leftrightarrow q = p'q' + pq = (p \oplus q)' = p' \Leftrightarrow q' = p' \oplus q = p \oplus q'$
- ~~$p \Leftrightarrow$~~ $p \oplus q = pq' + qp' = p' \Leftrightarrow q = p \Leftrightarrow q' = p' \oplus q'$
- $p \Leftrightarrow q = (p'+q)(p+q')$ $[(p \Rightarrow q) \wedge (q \Rightarrow p)]$
- $p \oplus q = p \oplus q'$
- $p' \oplus q = p \Leftrightarrow q = p \oplus q'$

- p: A number is even if and only if divisible by 2. $[p \Rightarrow q \wedge p \Leftarrow q]$
  P': A number is even or it is divisible by 2, but not both.

- NOR - Neither ... NOR
- OR - Either ..... OR

Negation for predicate-

$P(x)$	$\sim P(x)$
$\forall x P(x)$	$\exists x \sim P(x)$
$\exists x P(x)$	$\forall x \sim P(x)$
$\forall x \sim P(x)$	$\exists x P(x)$
$\exists x (\sim P(x))$	$\forall x P(x)$

$\sim(\forall x\,(P(x) \to Q(x))) \equiv \exists x(\sim[P(x) \to Q(x)])$

$\equiv \exists x(P(x) \wedge \sim Q(x))$

$\sim(\forall x\, \exists y\, P(x,y)) \equiv \exists x\, \forall y \sim P(x,y)$

$\sim(\exists x\, \forall y \forall z\, (P(x,y,z) \oplus Q(x,y,z))) \equiv \forall x\, \exists y\, \exists y\, (P(x,y,z) \Leftrightarrow Q(x,y,z))$

$\sim(p \Rightarrow q) = \sim(p'+q)$

$p \Rightarrow q$	[stmt]	$(p=0 \text{ or } q=0) \Rightarrow (pq=0)$
$q \Rightarrow p$	[converse]	$(pq \Rightarrow 0) \Rightarrow (p=0 \text{ or } q=0)$
$\sim p \Rightarrow \sim q$	[inverse]	$(p \neq 0 \text{ and } q \neq 0) \Rightarrow (pq \neq 0)$
$\sim q \Rightarrow \sim p$	[contrapositive]	$(pq \neq 0) \Rightarrow (p \neq 0 \text{ and } q \neq 0)$

inary operators-

p	q	$p+q$ $p \vee q$	$p \cdot q$ $p \wedge q$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$	$p \uparrow q$	$p \downarrow q$
0	0	0	0	1	1	0	1	1
0	1	1	0	1	0	1	1	0
1	0	1	0	0	0	1	1	0
1	1	1	1	1	1	0	0	0

if two prepositions are equivalent (X, Y)

then $[X \Leftrightarrow Y \equiv 1]$ $\quad [X \equiv Y \text{ iff } X \Leftrightarrow Y = 1]$

Q Let $b \Leftrightarrow c$ and $a \Leftrightarrow (b \vee \neg b)$ is tautology
what can be inferred about $a \vee (b \wedge c)$?

$b \Leftrightarrow c \Rightarrow b \equiv c$

$a \Leftrightarrow (b \vee \sim b) \Rightarrow a = 1$

$\therefore a \vee (b \wedge c) = a \vee (b \wedge b) = 1 \vee b = 1$ (Tautology)

oolean Alzebra: $(S, +, \cdot, ')$ $\quad$ [logic, Digital logic, Set theory]

$(S, \vee, \wedge, \sim)$

$(S, \cup, \cap, A^c)$

- No of elements in set of Boolean Alzebra must be in power of 2.
- Q_n is a boolean Alzebra.

- $a - b = a \wedge b'$
- $A - (B \cup C) = (A-B) \cup (A-C)$ (Test T or F)

$a - (b+c) = (a-b) + (a-c)$

$ab'c' = ab' + ac'$ (false)

Properties of Operators- • operators are also known as logical connectives.

1. <u>Closure-</u> $\forall x, y, \begin{bmatrix} x+y \in S \\ x \cdot y \in S \end{bmatrix}$ OR $\begin{bmatrix} x \vee y \in S \\ x \wedge y \in S \end{bmatrix}$

$\neg x \in S$ $\qquad \neg p \in S$

$\forall A, B \in S \begin{bmatrix} A \cup B \in S \\ A \cap B \in S \\ A^c \in S \end{bmatrix}$

2. <u>Commutative</u>:

$\forall x, y \in S \begin{bmatrix} x+y = y+x \\ x \cdot y = y \cdot x \end{bmatrix}$ $\qquad \forall A, B \in S \begin{bmatrix} (A \cup B) = (B \cup A) \\ (B \cap A) = (A \cap B) \end{bmatrix}$

$\forall x, y \in S \begin{bmatrix} x \wedge y = y \wedge x \\ x \vee y = y \vee x \end{bmatrix}$

3. <u>Associative</u>:

$\forall x, y, z \in S \begin{bmatrix} x+(y+z) = (x+y)+z \\ x \cdot (y \cdot z) = (x \cdot y) \cdot z \end{bmatrix}$

$\forall x, y, z \in S \begin{bmatrix} x \wedge (y \wedge z) = (x \wedge y) \wedge z \\ x \vee (y \vee z) = (x \vee y) \vee z \end{bmatrix}$

$\forall A, B, C \in S \begin{bmatrix} A \cup (B \cup C) = (A \cup B) \cup C \\ (A \cap B) \cap C = A \cap (B \cap C) \end{bmatrix}$

4. <u>Distributive</u>: $\forall x, y, z \in S \begin{bmatrix} x+(y \cdot z) = (x+y)(x+z) \\ x \cdot (y+z) = xy + xz \end{bmatrix}$

$\forall x, y, z \in S \begin{bmatrix} x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) \\ x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z) \end{bmatrix}$

$\forall \cancel{x, y, z} \; A, B, C \in S \begin{bmatrix} A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \end{bmatrix}$

5. <u>Identity</u>: $\begin{bmatrix} \exists_0 \forall x \;\; x+0 = x = 0+x \\ \exists_1 \forall x \;\; x \cdot 1 = x = 1 \cdot x \end{bmatrix}$ & $0 \neq 1$

$\forall x \in S \begin{bmatrix} x \wedge T = x = T \wedge x \\ x \vee F = x = F \vee x \end{bmatrix}$

$\exists_\phi \; \forall A \in S \begin{bmatrix} A \cup \phi = A = \phi \cup A \\ A \cap S = A = S \cap A \end{bmatrix}$ $\qquad [S \to \text{Universal set}]$
$\exists_S \; \forall A \in S$