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By-Ramesh sundaram sir

- Theory
- Explanation
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1. Logic
2. Combinatorics
3. Set Theory [KOLMAN, BUSAN & ROSS]
4. Graph Theory [NARSINGH DEO]

[Theory]

LOGIC

1. Logical Statement ?
[Proposition]
- 2. Logical Operators & their properties ($\vee, \wedge, \neg, \Rightarrow, \Leftrightarrow, \oplus, \uparrow, \downarrow$)
Disjunction
↑ Conjunction, Negation
3. Tautology^(T), Contradiction^(C) & Contingency (CT)
[Satisfiable/Unsatisfiable]
(T or CT) (C)
- 4. Normal Forms: PDNF (Principle Disjunctive Normal Form) & properties
PCNF (Principle Conjunctive Normal Form)
- 5. Implications & Biconditional ($\Rightarrow, \Leftrightarrow$)
6. Arguments & Fallacy [Invalid Argument]
7. Rules of Inference
- 8. Predicate Logic - Quantifier (\forall, \exists)

• Validity of a predicate
• Properties
• Translation

LOGIC

(S, O)
 ↓ ↗ operators
 Set of all Logical Stmt

Logical Statement - (Proposition)

• Declarative Sentence which can be either true or false but not both.

Ex - This board is white.

This Fan is Rotating
 • This sentence is true.

[is/will tends to] declaration

Not a Logical Statement

1. Questions - What is Your Name?
2. Command - Stand up.
3. Exclamation - Oh! That's great.
4. $x \neq 2 = 4$
 (it is not proposition bcoz for some x value it is true) it is false
5. He is tall. (unless he is specified)
6. Today is Wednesday.
 [Not a proposition bcoz today may be true, but tomorrow it will become false]
7. Tomorrow it will rain.
 [Not a proposition.]
8. This sentence is false.
 [Negative Self Referential Sentence]

Logical Operators :

A proposition is written in the following way:

$p: 2+2=4$

$q(x): x+2=4$ (Predicate) but not a proposition

False $\rightarrow \forall x P(x)$
 True $\rightarrow \exists x P(x)$] - propositions

Negation - (\neg, \bar{p}, \neg, p') • Unary operators

P	\bar{p}
0	1
1	0

P	Negation
is	is not
is not	is
=	\neq
<	\geq
>	\leq
$p \vee q$	$p' \wedge q' \Rightarrow p \vee q$
$p \wedge q$	$p' \vee q' \Rightarrow p \wedge q$

P	Negation
$p \Rightarrow q$	$p q'$
$p \Leftrightarrow q$	$p \oplus q$
$p \oplus q$	$p \Leftrightarrow q$
$p \wedge q$	$p \vee q$
$p \vee q$	$p \wedge q$

• if $p \vee q = 1$
 than $p = \neg q$ is one possibility but not the sure thing. it also allow some other thing

• if $p \wedge q = 0$
 $\Rightarrow [p = \neg q]$ not always.

• If $p \vee q = 1$ & $p \wedge q = 0 \Rightarrow [p = \neg q]$

• If $p \Rightarrow 2+2=4$ or $3+7=10$
 $\bar{p} \Rightarrow 2+2 \neq 4$ and $3+7 \neq 10$

• If $p \Rightarrow 2+2=4$ and $3+7=10$
 $\bar{p} \Rightarrow 2+2 \neq 4$ OR $3+7 \neq 10$

• p : 2 is even & divisible by 4.

p' : 2 is odd or not divisible by 4.

• p : if it rains, I will carry umbrella. [Either it does not rain OR I will carry umbrella]

p' : It rains and I will not carry umbrella

\wedge - all must be true
 \vee - atleast one is true
 \oplus - Exactly one is true
 \uparrow - Atleast one is false

Conversion of Secondary operators into Basic operators:

• $p \rightarrow q = p' + q$

• $p \Leftrightarrow q = p'q' + pq = (p \oplus q)' = p' \oplus q' = p' \oplus q = p \oplus q'$

• $p \Rightarrow q = pq' + qp' = p' \oplus q = p \oplus q' = p' \oplus q'$

• $p \Leftrightarrow q = (p' + q)(p + q')$ $[(p \Rightarrow q) \wedge (q \Rightarrow p)]$

• $p \oplus q = p \oplus q'$

• $p' \oplus q = p \Leftrightarrow q = p \oplus q'$

• p : A number is even if and only if divisible by 2. $[p \Rightarrow \bar{q} \wedge p \Leftarrow q]$

p' : A number is even or it is divisible by 2, but not both.

• NOR - Neither ... NOR

• DR - Either ... OR

Negation for predicate-

$P(x)$	$\neg P(x)$
$\forall x P(x)$	$\exists x \neg P(x)$
$\exists x P(x)$	$\forall x \neg P(x)$
$\forall x \neg P(x)$	$\exists x P(x)$
$\exists x (\neg P(x))$	$\forall x P(x)$

$$\neg(\forall x(P(x) \rightarrow Q(x))) \equiv \exists x(\neg(P(x) \rightarrow Q(x)))$$

$$\equiv \exists x(P(x) \wedge \neg Q(x))$$

$$\neg(\forall x \exists y P(x, y)) \equiv \exists x \forall y \neg P(x, y)$$

$$\neg(\exists x \forall y \forall z (P(x, y, z) \oplus Q(x, y, z))) \equiv \forall x \exists y \exists z (P(x, y, z) \oplus Q(x, y, z))$$

$$\neg(p \Rightarrow q) = \neg(p \wedge \neg q)$$

$p \Rightarrow q$	[stmt]	$(p=0 \text{ or } q=0) \Rightarrow (pq=0)$
$q \Rightarrow p$	[converse]	$(pq \neq 0) \Rightarrow (p=0 \text{ or } q=0)$
$\neg p \Rightarrow \neg q$	[inverse]	$(p \neq 0 \text{ and } q \neq 0) \Rightarrow (pq \neq 0)$
$\neg q \Rightarrow \neg p$	[contrapositive]	$(pq \neq 0) \Rightarrow (p \neq 0 \text{ and } q \neq 0)$

major operators-

P	Q	$p+q$	$p \cdot q$	$p \vee q$	$p \wedge q$	$p \Rightarrow q$	$p \Leftrightarrow q$	$p \oplus q$	$p \uparrow q$	$p \downarrow q$
0	0	0	0	0	0	1	1	0	1	1
0	1	1	0	1	0	1	0	1	1	0
1	0	1	0	1	0	0	0	1	1	0
1	1	1	1	1	1	1	1	0	0	0

if two propositions are equivalent (x, y)

$$\text{then } [x \Leftrightarrow y \equiv 1] \quad [x \equiv y \text{ iff } x \Leftrightarrow y = 1]$$

\exists let $b \Leftrightarrow c$ and $a \Leftrightarrow (b \vee \neg b)$ is tautology
what can be inferred about $a \vee (b \wedge c)$?

$$b \Leftrightarrow c \Rightarrow b \equiv c$$

$$a \Leftrightarrow (b \vee \neg b) \Rightarrow a = 1$$

$$\therefore a \vee (b \wedge c) = a \vee (b \wedge b) = 1 \vee b = 1 \text{ (Tautology)}$$

Boolean Algebra: $(S, +, \cdot, ')$

(S, \vee, \wedge, \neg)

(S, U, \cap, A^c)

[logic, Digital logic, Set theory]

• No. of elements in set of Boolean Algebra must be in power of 2.

• \mathbb{Q}_n is a Boolean Algebra.

$$a - b = a \wedge b'$$

$$A - (B \cup C) = (A - B) \cap (A - C) \text{ (Test T or F)}$$

$$a - (b + c) = (a - b) \cap (a - c)$$

$$a \wedge b' = a \wedge b' + a \wedge c' \text{ (false)}$$

Properties of Operators - Operators are also known as logical connectives.

1. Closure - $\forall x, y, \begin{bmatrix} x+y \in S \\ x \cdot y \in S \\ \sim x \in S \end{bmatrix}$ OR $\begin{bmatrix} x \vee y \in S \\ x \wedge y \in S \end{bmatrix}$

$\forall A, B \in S \begin{bmatrix} A \cup B \in S \\ A \cap B \in S \\ A^c \in S \end{bmatrix}$

2. Commutative:

$\forall x, y \in S \begin{bmatrix} x+y = y+x \\ x \cdot y = y \cdot x \end{bmatrix}$

$\forall A, B \in S \begin{bmatrix} (A \cup B) = (B \cup A) \\ (A \cap B) = (B \cap A) \end{bmatrix}$

$\forall x, y \in S \begin{bmatrix} x \wedge y = y \wedge x \\ x \vee y = y \vee x \end{bmatrix}$

3. Associative:

$\forall x, y, z \in S \begin{bmatrix} x+(y+z) = (x+y)+z \\ x \cdot (y \cdot z) = (x \cdot y) \cdot z \end{bmatrix}$

$\forall x, y, z \in S \begin{bmatrix} x \wedge (y \wedge z) = (x \wedge y) \wedge z \\ x \vee (y \vee z) = (x \vee y) \vee z \end{bmatrix}$

$\forall A, B, C \in S \begin{bmatrix} A \cup (B \cap C) = (A \cup B) \cap C \\ (A \cap B) \cup C = A \cap (B \cup C) \end{bmatrix}$

4. Distributive:

$\forall x, y, z \in S \begin{bmatrix} x+(y \cdot z) = (x+y) \cdot (x+z) \\ x \cdot (y+z) = xy + xz \end{bmatrix}$

$\forall x, y, z \in S \begin{bmatrix} x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) \\ x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z) \end{bmatrix}$

$\forall x, y, z \in S \begin{bmatrix} A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \end{bmatrix}$

5. Identity:

$\begin{bmatrix} \exists 0 \forall x \ x+0 = x = 0+x \\ \exists 1 \forall x \ x \cdot 1 = x = 1 \cdot x \end{bmatrix} \quad 0 \neq 1$

$\forall x \in S \begin{bmatrix} x \wedge F = x = T \wedge x \\ x \vee F = x = F \vee x \end{bmatrix}$

$\exists \phi \forall A \in S \begin{bmatrix} A \cup \phi = A = \phi \cup A \\ A \cap S = A = S \cap A \end{bmatrix}$

[S \rightarrow Universal Set]