MADE EASY

ESE /GATE/PSU

MATHMATICS

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- Theory
- Explanation
- Derivation
- Example
- Shortcuts
- Previous Years Question With Solution

MATHEMATICS

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GATE → 13-15 M

ESE → 15 Questions

Syllabus:

1. Linear Algebra
2. Probability
3. Calculus
4. Vector Calculus
5. Differential Equation
6. Complex number
7. Numerical Methods.
8. Laplace Transform.
9. Fourier Series

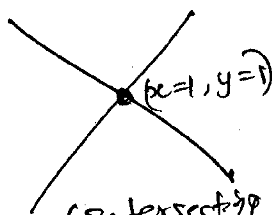
* LINEAR ALGEBRA :

study of Linear system of equation :

$x+2y=3 \rightarrow (1)$

$2x+3y=5 \rightarrow (2)$

$\Downarrow$



(Intersecting Line)

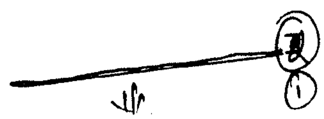
$\Downarrow$

Unique soln

$x+2y=3 \rightarrow (1)$

$2x+4y=6 \rightarrow (2)$

$\Downarrow$



$\Downarrow$

Coincident Line

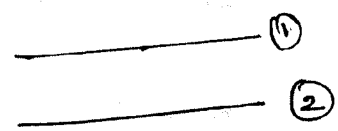
$\Downarrow$

Infinite soln

$x+2y=3 \rightarrow (1)$

$x+2y=5 \rightarrow (2)$

$\Downarrow$



$\Downarrow$

parallel line

$\Downarrow$

No solution.

If there are more than two variables

$\Downarrow$

we cannot plot graph and know about the soln.

$\Downarrow$

$\therefore$ To get soln $\Rightarrow$ we find Rank

$\Downarrow$

$\therefore$ we study **Matrix**, in Linear Algebra

Matrix :

$A = [a_{ij}]_{m \times n}$

$m \to$ No. of row $\to$ horizontal

$n \to$ No. of column $\to$ vertical

$a_{ij} \Rightarrow$ Element of matrix

$i \to$ row $j \to$ column

R → Row
C → column

$$\begin{array}{c} \\ A = \end{array}
\begin{array}{cccc|l}
a_{11} & a_{12} & \cdots & a_{1n} & \to R_1 \\
a_{21} & a_{22} & \cdots & a_{2n} & \to R_2 \\
\vdots & \vdots & & \vdots & \\
a_{m1} & a_{m2} & \cdots & a_{mn} & \to R_m \\
\downarrow & \downarrow & & \downarrow & m \times n \\
C_1 & C_2 & & C_n &
\end{array}$$

(i) If $m \neq n$ $\Rightarrow$ Rectangular Matrix.

(ii) If $m = n$ $\Rightarrow$ square matrix
$\Downarrow$
Diagonal element exist only in square matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

→ principal diagonal
main diagonal
leading diagonal
primary diagonal
diagonal elements

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$\boxed{\text{Trace of } A = \text{Sum of main diagonal elements}}$

$\Downarrow$

$Tr(A) = \sum a_{ij} \ \forall \ i = j$

(i) for diagonal element $\Rightarrow i=j \;\forall\, i,j$

(ii) for lower diagonal element $\Rightarrow i>j \;\forall\, i,j$

(iii) for upper diagonal element $\Rightarrow i<j \;\forall\, i,j$

(iv) for other than diagonal element $\Rightarrow i \neq j, \;\forall\, i,j$

(or)

off diagonal element

(v) corresponding element $= a_{ij}$ & $a_{ji} \;\forall\, i,j$

Ex: a_{31} & a_{13}

a_{23} & a_{32}

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* **Diagonal Matrix** :

All off diagonal element $=0$ & Atleast one diagonal element must not be zero

eg: $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, $\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ non-zero

where a, b, c are non-zero.

pb: Minimum no-of-zeroes in diagonal matrix of order 'n' ?

Minimum No-of zeroe's = total no-of element − no-of primary diagonal element

$= (n \times n) - n$

$= n^2 - n = \boxed{n(n-1)}$

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∴ Minimum no-of zeroes $= n^2 - n$
Max no-of zeroes $= n^2 - 1$

**

* Identity Matrix :

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

I_n = Identity matrix of order n

* Scalar Matrix :

$\boxed{A = K \cdot I}$

Ex: $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} = 7I_3$

Note : All Scalar Matrix are Diagonal matrix but All Diagonal matrix are Not scalar matrix

* Upper Triangular Matrix : (UTM)

$A = [a_{ij}]_{m \times n} \Rightarrow \boxed{a_{ij} = 0 \ \forall \ i > j}$

Ex: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$

* Lower Triangular Matrix (LTM) :

$A = [a_{ij}]_{m \times n} \Rightarrow \boxed{a_{ij} = 0 \ \forall \ i < j}$

Ex: $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$

* Column matrix : (Column vector)

$A = [a_{ij}]_{n \times 1}$

Ex: $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}_{4 \times 1}$

ONLY ONE COLUMN

* Row matrix : (Row Vector)

$A = [a_{ij}]_{1 \times n}$

$A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1 \times 4}$

ONLY ONE ROW

* Transpose matrix (A^T)

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

* Symmetric Matrix :

* $A^T = A$
* $a_{ij} = a_{ji}$

Ex: $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

* Skew-symmetric matrix:

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$$\boxed{\begin{array}{l} A^T = -A \\ a_{ij} = -a_{ji} \;\forall\, i \neq j \end{array}}$$

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -5 \\ 3 & 5 & 0 \end{bmatrix}$$

★ ALL LEADING DIAGONAL ELEMENTS MUST BE ZERO

$$A^T = -\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 5 \\ -3 & -5 & 0 \end{bmatrix} = -A$$

Note:

Sum of all elements of skew symmetric matrix = ZERO

Ex: $A = \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix}$; $A^T = \begin{bmatrix} 2 & 6 \\ 5 & 8 \end{bmatrix}$

$$\frac{A+A^T}{2} = \frac{1}{2}\begin{bmatrix} 4 & 11 \\ 11 & 16 \end{bmatrix} = \begin{bmatrix} 2 & 11/2 \\ 11/2 & 8 \end{bmatrix} \rightarrow (1) \rightarrow \text{symmetric matrix}$$

$$\frac{A-A^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 \\ 1/2 & 0 \end{bmatrix} \rightarrow (2) \rightarrow \text{skew symmetric matrix}$$

$$(1)+(2) \Rightarrow \begin{bmatrix} 2 & 5 \\ 6 & 8 \end{bmatrix} = A$$

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$$\therefore \boxed{A = \left(\frac{A+A^T}{2}\right) + \left(\frac{A-A^T}{2}\right)}$$

Any square matrix = symmetric matrix + skew symmetric matrix

Note:

Every square matrix can be expressed as the sum of symmetric & skew-symmetric matrix.

* Singular Matrix:

$|A| = 0$

* Non Singular matrix

$|A| \neq 0$

* Invertible matrix

$\Downarrow$

$\boxed{A^{-1} \text{ exist}}$

$$A^{-1} = \frac{Adj^{\prime}A}{|A|}$$

A^{-1} exist only when $|A| \neq 0$ (i.e Non-Singular)

* Complex matrix :

$$A = \begin{bmatrix} 1+i & 3-2i \\ 2+i & 5 \end{bmatrix}$$

conjugate of A $\Downarrow$ $\bar{A}$ $= \begin{bmatrix} 1-i & 3+2i \\ 2-i & 5 \end{bmatrix}$

Conjugate transpose $(\bar{A})^T$ (or) A^θ $= \begin{bmatrix} 1-i & 2-i \\ 3+2i & 5 \end{bmatrix}$

* Hermitian matrix .

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$(\bar{A})^T = A$

$a_{ij} = \overline{a_{ji}}$

**

Main diagonal Element are "purely Real"

ex:

$$A = \begin{bmatrix} 4 & 1+i \\ 1-i & 5 \end{bmatrix}$$

* Skew-Hermitian Matrix:

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$(\bar{A})^T = -A$

$a_{ij} = -\overline{a_{ji}}$

**

Main diagonal Element are "zero (or) purely Imaginary"

Ex:

$$A = \begin{bmatrix} i & -1+i \\ 1+i & i \end{bmatrix}$$

* operation of matrix :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

* Addition :

$$A+B = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} = B+A$$

$\boxed{A+B = B+A}$ ← Cummulative (two matrix)

$\boxed{A+(B+C) = (A+B)+C}$ ← Associative (three matrix)

Addition operation are cummulative & Associative

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* Subtraction :

$$A-B = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix} \quad ; \quad B-A = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$$

$\boxed{A-B \neq B-A}$

Substraction is Neither cummutative Nor Associative

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* Scalar Multiplication

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B = kA \Rightarrow B = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

k ↓ scalar